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LETTER TO THE EDITOR

The anisotropic correlation in percolation theory

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Abstract. The problem of anisotropic correlation in percolation theory is studied. The surface of the percolation thresholds x_c of the two-dimensional square lattice of sites 32×32 in the space $k_\parallel$, $k_\perp$, x_c is plotted by means of the Monte Carlo method, where $k_\parallel$ and $k_\perp$ are the anisotropic correlation parameters.

The recent advent of a great number of different models in percolation theory is due to the success which is achieved when applying the results obtained on these models to real physical systems. Since the previously suggested random theory of percolation (Broadbent and Hammersley 1957) could not describe the total variety of physical properties of the systems under investigation, its development has proceeded mainly in two directions. The first was connected with the problems of taking into account the interaction within the system, which lead to the appearance of the percolation threshold value dependence on the correlation parameter, characterising this interaction (Duckers and Ross 1974, Napiorkowski and Hemmer 1980). Another approach was concerned with the difficulties arising from considering the problem of random percolation within the systems with anisotropic properties (Guyon 1981, Sarichev and Vinogradov 1983). At the same time, there are physical processes requiring, in the course of their description, a simultaneous consideration of the correlation effect and anisotropy. Thus, when considering the process of the disperse destruction within the framework of percolation theory (Chelidze 1983, Chelidze and Kolesnikov 1983), it becomes necessary to take into account the effect of the local stress field of elementary fracture on the probability of the new fracture occurrence in its vicinity. At the same time, it should be noted that this probability may considerably differ in different directions due to the effect of external forces or due to the anisotropy characteristic of the material, which finally leads to the occurrence of the anisotropy correlation (AC) within the system under consideration. The AC effect on the destruction process development can be revealed by analysis of microphotographs of the crack networks arising in the samples with their deformation.

However, since the destruction process description is not the main point of this letter, we will proceed to formulate the percolation problem within AC systems, using the conventional and general terms of percolation theory for the simplest system—the square lattice of sites. (We note herewith that the problem of percolation within the AC systems comes close to the Ising model with anisotropic interaction between the spins considered in the phase transition theory.)

Let the population probability of the vacant lattice site depend on the condition of its four nearest-neighbour sites, and let x be the population probability of the vacant

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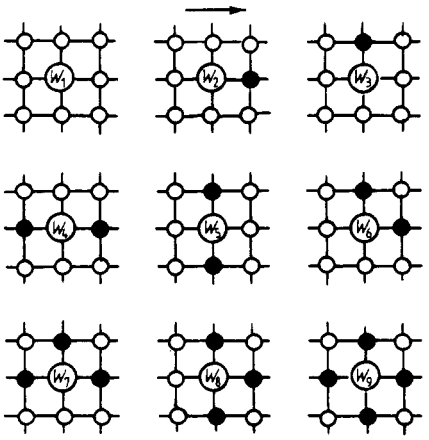

Figure 1. Possible arrangements of vacant sites in the square lattice and corresponding probabilities W_i (full circles denote occupied sites, the arrow is oriented in the $k_\parallel$ direction).

site, having no neighbouring populated sites, and k is the parameter of correlation, showing how many times the population probability of the vacant site having a neighbouring populated site is higher or lower than x. To introduce AC, we determine two different correlation parameters $k_\parallel$ and $k_\perp$ for two perpendicular directions. Then, as shown in figure 1, the vacant sites with nine different population probabilities may be present in the lattice. Since the total population probability should sum to unity for all vacant lattice sites, we have

$$\sum_{i=1}^{9} n_i W_i = 1 \tag{1}$$

where n_i is a number of sites with the population probability

$$W_i(W_1 = x,\ W_2 = k_\parallel x,\ W_3 = k_\perp x,\ W_4 = k_\parallel^2 x,\ W_5 = k_\perp^2 x,$$
$$W_6 = k_\parallel k_\perp x,\ W_7 = k_\parallel^2 k_\perp x,\ W_8 = k_\parallel k_\perp^2 x,\ W_9 = k_\parallel^2 k_\perp^2 x).$$

It is clear that when populating the lattice, the pattern of probability distribution over the vacant sites changes, and new probabilities may be calculated (1) allowing for the changes of n_i. In order to study the functional dependence $x_c = F(k_\parallel, k_\perp)$ the AC-percolation modelling program was set up and realised on the EC-1055 type computer. Let us now turn to the results of computation experiments which may be represented as the percolation threshold surfaces in space $\log k_\parallel$, $\log k_\perp$, $x_c(k_\parallel, k_\perp)$.

Figure 2 represents this surface for the 32×32 square lattice. The cross marks the point which corresponds to the random model. A broken curve on the surface—the result obtained earlier by Duckers and Ross (1974)—corresponds to the problem with correlation factor over unity. The extreme left point of the surface under consideration corresponds to the repulsive type of interaction (the correlation factor is smaller than unity) discussed in Napiorkowski and Hemmer (1980). Two regions with a stable character of the system behaviour at the correlation parameters variation may be marked out. In the region of parameters $\log k_\parallel < 0$ the type of x_c dependence on $\log k_\perp$ is similar to the type of x_c dependence on $\log k_\parallel$ at $\log k_\perp < 0$. Part of the surface limited by $\log k_\parallel$, $\log k_\perp > 0$ is characterised by a sharp change in x_c for a small change in $\log k_\parallel / k_\perp$. For example, at a slight $\log k_\parallel / k_\perp$ variation in this region, x_c rolls down

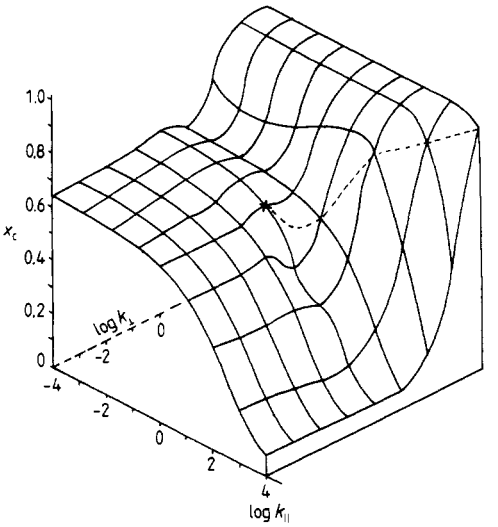

Figure 2. The surface of percolation threshold in $(x_c, \log k_\parallel, \log k_\perp)$-space for the 32×32 square lattice.

from the 'saturation plateau', where $x_c \to 1$, into the 'zero valley' with $x_c \to 0$. It should be mentioned that our results were obtained at finite lattices. Naturally, the surface character will be preserved with the increase in the lattice size, though some of its details would vary. Depending on the relation between the system sizes and the degree of correlation, the percolation process will progress, either uniformly and dispersely in the entire system (all basic regularities of the classical percolation theory being preserved), or this process becomes so strongly localised that it cannot be described by percolation theory.

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